# MOTION OF A CONDUCTIVE PISTON IN A CHANNEL WITH VARIABLE INDUCTANCE

V. V. Polyudov, V. M. Titov,
and G. A. Shvetsov

UDC 533.9 : 533.95

The motion of a conductive piston in the channel of a magnetohydrodynamic (MHD) generator of the conduction type with compound electrodes is considered. Formulas are obtained for calculation of the energy characteristics of the pulse MHD generator for various operational regimes. It is shown that in an MHD generator at magnetic Reynolds number values $Re_m = \mu_0 \sigma u_0 l \gg 1$ (where $\mu_0$ is the permeability of a vacuum, $\sigma$ is the electrical conductivity of the piston, $u_0$ is the initial velocity, and $l$ is the characteristic dimension), the energy transferred to an ohmic load may significantly exceed the values obtained in [1, 2]. Conditions for high-efficiency transformation of piston kinetic energy to electrical energy are considered for limiting values of the ratio of the latter to initial magnetic field energy in the generator channel.

In [1, 2] an analysis was performed of the energy characteristics of impulse MHD generators with $Re_m \gg 1$.

In [1] it was maintained that the change in magnetic field in the MHD generator channel with motion of a plasma with $Re_m \gg 1$ leads to a maximum value of the braking pressure $2B_0^2/\mu_0$ ($B_0$ is the initial magnetic field).

It follows from this that the power and energy are then limited respectively to $4W_0u_0/l_0$ and $4W_0$ ($W_0$ is the initial magnetic field energy and $l_0$ is the generator length).

In [2], in analysis of a magnetocumulative generator with ohmic load it is concluded that the energy supplied to an active load $R_1$ = const has a maximum equal to $W_0 \ln N$, where $N = L_0/L_1$ is the ratio of the initial generator channel inductance to the load inductance.

The conclusions of [1, 2] require refinements, as cited, for example, in [3-6], and there is no detailed analysis of the energy characteristics of impulse MHD generators in the literature. This present study will consider the basic energy relationships in impulse MHD systems using a simple electrotechnical model.*

1. We shall consider the operation of a linear MHD generator, with electrodes connected to an inductive-resistive load. These electrodes are switched to the primary energy source, a capacitor bank, and at the moment of maximum discharge current $I_0$ a moving conductive piston shorts the electrodes; the primary energy source is switched out of the circuit and will not be considered further. The inductance of the circuit at the switching time is $L_0$, and for an arbitrary moment of time

$$L(t) = \begin{cases} L_0 - \int\limits_0^t \dot{L}dt & (t \in [0; T]) \\ L_1 + L_2 & (t \geqslant T) \end{cases}$$

*V. S. Sokolov has called attention to the errors in the conclusion of [1].

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 41-46, November-December, 1973. Original article submitted July 31, 1972.

where T is determined by the equation

$$\int_0^T u(t)\,dt = l_0$$

$\dot{L}$ is the time rate of change of inductance, $L_2$ is the internal inductance of the generator, and $L_1$ is the load inductance.

The sum of the voltages and the energy balance for such a circuit have the form

$$\frac{d}{dt}(L(t)I(t)) + I(t)R = 0 \qquad (R = R_1 + R_2) \tag{1.1}$$

$$\frac{L_0 I_0^2}{2} + (E(0) - E(t)) = \frac{LI^2}{2} + \int_0^t I^2 R\,dt = W + \varepsilon \tag{1.2}$$

where E is the kinetic energy of the moving piston, $R_2$ is the internal resistance of the generator, and $R_1$ is the load resistance.

In these expressions $R_1$, $R_2$, $\dot{L}$ vary with time and are functions of the energy generated and the piston position within the generator. Therefore attaining solutions of Eqs. (1.1), (1.2) for the general case in the form of an exact expression in elementary functions is impossible. However, if it is assumed that the quantities mentioned are constant, Eq. (1.1) admits a simple solution and W and ε may be calculated.

Assuming

$$\dot{L} = \text{const}, \quad R_1 = \text{const}, \quad R_2 = \text{const} \tag{1.3}$$

we find for $t \in [0; T]$

$$I = I_0 N(t)^{1-1/\gamma} \tag{1.4}$$

where $N(t) = L_0/L(t)$ is the coefficient of circuit inductance change, and $\gamma = \dot{L}/R$.

For $t > T$ the circuit does not change ($\dot{L} = 0$) and the current is found from the equation

$$L_* dI/dt + IR = 0$$

$$I = I_* \exp[-RL_*^{-1}(t - T)] = I_0 N_*^{1-1/\gamma} \exp[-RL_*^{-1}(t - T)] \tag{1.5}$$

The values of L, I, N, and W at time T are denoted by $L_*$, $I_*$, $N_*$, $W_*$.

It follows from Eq. (1.4) that the current in the circuit increases for $\gamma > 1$ and decreases for $\gamma < 1$, while the rate of decrease increases for $\gamma \to 0$.

For $t \in [0, T]$ let $\gamma > 1$, in which case three segments may be delineated in the curve of current with respect to time. The first phase up to t = 0 is determined by the primary energy source, which produces the current $I_0$.

In the second phase from 0 to T the current in the circuit increases, and in the third phase for $t > T$ the current decays exponentially.

The energy supplied to the load $R_1$ after the piston shorts the generator electrodes will be

$$\varepsilon' = \varepsilon_2' + \varepsilon_3'$$

The indices 2, 3 denote the respective phases of the process

$$\varepsilon_2' = \int_0^T I_2^2 R_1\,dt = \begin{cases} W_0 \dfrac{2k}{\gamma - 2}(N_*^{1-2/\gamma} - 1) & (\gamma \neq 2) \\ kW_0 \ln N_* & (\gamma = 2) \end{cases} \tag{1.6}$$

$$\varepsilon_3' = \int_T^{T+t_3} I_3^2 R_1 dt = kW_*'\left[1 - \exp\left(-\frac{2R}{L_*}t_3\right)\right] \quad (k = R_1/(R_1 + R_2),\ W_0 = L_0 I_0^2/2) \tag{1.7}$$

$$W_*' = W_0 N_*^{1-2/\gamma} \tag{1.8}$$

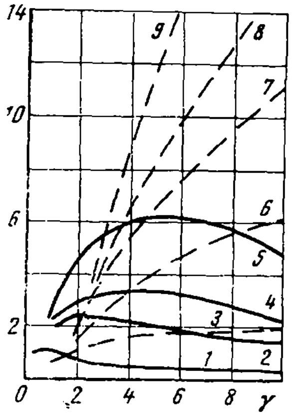


Fig. 1

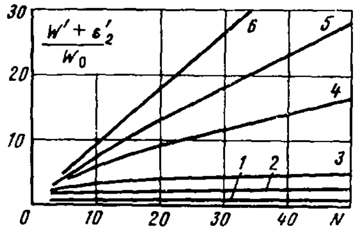


Fig. 2

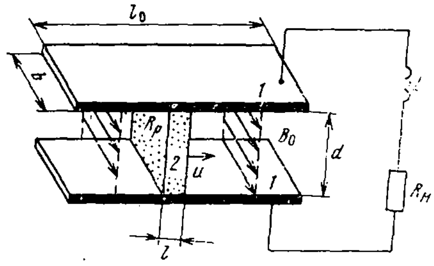


Fig. 3

where $W_*'$ is the magnetic field energy at t = T, and k is the load coefficient.

It is interesting that for arbitrary variation of $\dot{L}'(t)$ and R(t), the values of $I_*$, $\varepsilon_2'$ and $W_*'$ are still determined by Eqs. (1.4), (1.6), (1.8) if

$$\dot{L}(t)/R(t) = \gamma = \text{const} \quad t \in [0;\ T]$$

Figure 1 shows curves illustrating the dependence of $\varepsilon_2'/W_0$ (solid line) and $W_*'/W_0$ (dashes) on $\gamma$ for given $N_*$ and k = 1, while Fig. 2 shows curves of $(W_*' + \varepsilon_2')/W_0$ versus $N_*$ for given $\gamma$ and k = 1.

In Fig. 1 each pair of curves (1, 3), (2, 6), (4, 7), and (5, 8) corresponds to $N_* = 2, 10, 20, 50$.

In Fig. 2, curves 1, 2, 3, 4, 5, 6 correspond to $\gamma$ equal to 0.1, 1, 2, 5, 10, 50.

From Eqs. (1.6)-(1.8) and Figs. 1, 2 it follows that for $\gamma \le 1$ the system energy does not exceed $2W_0$. For $\gamma \le 2$ $\varepsilon_2' \le W_0 \ln N_*$, $W_*' \le W_0$.

For a given $N_*$ there exists a $\gamma$ at which $\varepsilon_2'$ is maximum, while for $\gamma > 2$, beginning at some $N_*$ this maximum may significantly exceed $W_0 \ln N_*$. For $\gamma \gg 2$ the system energy increases basically due to magnetic field energy. The increase in energy is then limited by the ratio of the initial inductance to the inductance $L_*$. We will now assume $L_1 \gg L_2$. For $R_1 \gg R_2$ an energy $W_*'$ may be transferred to the load if the energy extraction time in the third phase is greater than the relaxation time of the circuit $L_1$, $R_1$:

$$t_3 \gg L_1/R_1 = \tau \tag{1.9}$$

In this case $\varepsilon_3' \approx W_*'$ and the total energy transferred to the load $R_1$ = const is $\varepsilon' \approx \varepsilon_2' + W_*$.

2. We will consider the motion of a conductive piston in an MHD channel of constant cross section, located in an external magnetic field $B_0$ (1-electrodes, 2-conductive piston in Fig. 3). As before, we assume the conditions of Eq. (1.3) to be fulfilled. The sum of the voltages for this case will be

$$\frac{d}{dt}(LI) + IR = u_0 B_0 d \tag{2.1}$$

With consideration of the initial conditions $(I|_{t=0} = 0)$ for $t \in [0;\ T]$ we obtain

$$I = \begin{cases} B_0 b \mu_0^{-1} \ln N(t) & (\gamma = 1) \\ \dfrac{B_0 b}{\mu_0} \dfrac{1}{1 - 1/\gamma} \left(N_{(t)}^{1-1/\gamma} - 1\right) & (\gamma \neq 1) \end{cases} \tag{2.2}$$

For t > T

$$I = I_* \exp\left(-R L_*^{-1} (t - T)\right) \tag{2.3}$$

The energy transferred to the active load $R_1$ is

$$\varepsilon'' = \int_0^T I_2^2 R_1\, dt + \int_T^{T+t_3} I_3^2 R_1\, dt = \varepsilon_2'' + \varepsilon_3''$$

(indices 2, 3 are used for uniformity of notation)

$$\varepsilon_2'' = \begin{cases} 4W_0 k \left(1 - \dfrac{\ln^2 N_* + 2\ln N_* + 2}{2N_*}\right) & (\gamma = 1) \\ 4W_0 k \left[1 - 4\left(1 - \dfrac{1}{N_*^{1/2}}\right) - \dfrac{1}{N_*} + \ln N_*\right] & (\gamma = 2) \\ \dfrac{2W_0 k\gamma}{(\gamma-1)^2}\left[\dfrac{\gamma}{\gamma-2}\left(N_*^{1-2/\gamma} - 1\right) - 2\gamma\left(1 - \dfrac{1}{N_*^{1/\gamma}}\right) + \left(1 - \dfrac{1}{N_*}\right)\right]_{\gamma\neq 2}^{\gamma\neq 1} & \end{cases} \tag{2.4}$$

$$\varepsilon_3'' = kW_*'' \left[1 - \exp\left(-2RL_*^{-1} t_3\right)\right] \tag{2.5}$$

$$W_*'' = \begin{cases} W_0 N_*^{-1} \ln^2 N & (\gamma = 1) \\ \dfrac{W_0 \gamma^2}{(\gamma-1)^2}\left[N_*^{1-2/\gamma} - 2N_*^{-1/\gamma} + 1/N_*\right] & (\gamma \neq 1) \end{cases} \tag{2.6}$$

where $W_0 = B_0^2 V_0/2\mu_0$ is the initial magnetic field energy, and $V_0$ is the useful volume of the MHD channel.

Figure 4 shows the dependence of $\varepsilon_2''/W_0$ (solid line) and $W_*''/W_0$ (dashes) on $\gamma$ for given $N_*$ and $k = 1$. For $\varepsilon_2''$, just as for $\varepsilon_2'$, a maximum is characteristic, although attained at lower $\gamma$ values. In absolute value $\varepsilon''_{2max}$ insignificantly exceeds $\varepsilon'_{2max}$.

If $t_3$ satisfies the condition of Eq. (1.9) and $R_1 \gg R_2$, then $\varepsilon_3'' \approx W_*''$ and the total energy supplied to the active load $R_1$ = const is equal to $\varepsilon'' = \varepsilon_2'' + W_*''$.

Thus, in the methods considered to obtain a maximum ratio $\varepsilon/W_0$ it is first necessary to achieve transformation of the kinetic piston energy into electromagnetic energy at the maximum possible $\gamma$, and then at minimum possible $\gamma$ (with limit $\gamma = 0$) to transfer this energy to the load. Since

$$\gamma = L/R = (1 - K)\,\mu_0 \sigma u_0 \delta = (1 - k)\,\mathrm{Re}_m\, \delta/l$$

all the values defined in Eqs. (1.4)-(1.8), (2.2)-(2.6) can be considered as functions of the magnetic Reynolds number $\mathrm{Re}_m$, calculated from the piston length $l$ ($\delta$ is the effective depth of the current layer in the piston, $\delta \leq l$).

The analysis presented shows that in an impulse MHD generator with $\gamma \gg 1$ ($\mathrm{Re}_m = \gamma(1-k)^{-1}\, l/\delta \gg 1$) the energy transferred to an active load may significantly exceed the values obtained in [1, 2].

For $\mathrm{Re}_m \ll 1$ the formulas for current (2.2) and energy (2.4) transform to the well-known expressions

$$I \leqslant B_0 b \mu_0^{-1} (1 - k)\,\mathrm{Re}_m$$

$$\varepsilon = 2W_0 k (1 - k)\,\mathrm{Re}_m = Pl_0/u_0 \qquad (P = \sigma u_0^2 B_0^2 k (1 - k)\, bdl)$$

where P is the power in the MHD generator load.

3. In real generators, as was noted above, $\dot{L} \neq$ const, and so achieving solutions of Eqs. (1.1), (2.1) in complete form is not possible. However, for calculation, the energy conversion process can be represented as the sum of time intervals, during each of which $\dot{L}$ = const. In each interval the formulas for current and energy are analogous to those obtained above. It is evident that in systems in which $\dot{L}$ is a decreasing function, for one and the same $R_1$, $R_2$, and $N_*$

$$\frac{\varepsilon[\gamma(0)]}{W_0} > \frac{\varepsilon[\gamma(t)]}{W_0} > \frac{\varepsilon[\gamma(T)]}{W_0}$$

In the general case $\dot{L}$ is a function of piston speed and the geometrical dimensions of the generator channel. For a generator of constant cross section, the condition $\dot{L}$ = const is equivalent to the condition that $u(t) \approx u_0$ = const.

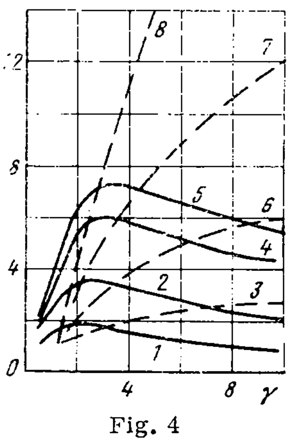

Fig. 4

It is evident that if the piston speed during the energy conversion process is almost constant, only a relatively small fraction of the kinetic energy will be transformed into electrical, and so the useful-work coefficient of the generator

$$\eta = (E_0 - E_*)/E_0$$

will be low. With decrease in piston speed $\eta$ increases, but this leads to decrease in $\dot{L}$ and finally to decrease in $\varepsilon/W_0$. The problem of simultaneous maximization of $\eta$ and $\varepsilon/W_0$ will not be considered here. We simply note that this requirement may be achieved if the reduction in piston speed is compensated by a corresponding change in the geometrical dimensions of the generator channel.

For transfer of energy $W_*$ to an active load the condition of Eq. (1.9) is necessary. It may be satisfied in several manners. Let us assume that the piston shorts the circuit $L_1R_1$, having reached the ends of the electrodes. If $l/u_* \geq \tau$, then $\varepsilon_3 \approx W_*$. If the circuit $L_1$, $R_1$ is closed for a period of time $t_3 < L_1'/R_1$, the energy $W_*$ may be transferred to the load by artificially reducing the system relaxation time. For example, the load resistance may be increased sharply [2, 7]. The condition $t_3 \geq \tau R_1/R_3$ will then determine the necessary resistance increase.

The formulas presented in this study permit evaluation of the basic energy characteristics and analysis of the operation of impulse energy converters.

The authors thank V. I. Yakovlev for his helpful evaluation.

## LITERATURE CITED


1. H. J. Pain and P. R. Smy, "Experiments on power generation from a moving plasma," J. Fluid Mech., 10, No. 1 (1961).
2. R. L. Conger, "Large electric power pulses by explosive magnetic field compressions," J. Appl. Phys., 38, No. 5 (1967).
3. M. Jones, C. MacKinnon, and V. Bleckman, "Generation of short-duration pulses in linear MHD generators," in: Applied Magnetohydrodynamics [Russian translation], Mir, Moscow (1965).
4. J. Bernard, "Conversion d'energie explosive en energie electromagnetique," Bibliographie CEA, Nr. 85 (1967).
5. M. S. Jones Jr. and V. H. Bleckman, "Parametric studies of explosive-driven MHD power generators," Proc. Internat. Sympos. Magnetohydrodynamic Electric Power Generation, 1964, Vol. 2, Paris (1964).
6. Conger, Johnson, Long, and Parks, "Generation of powerful electrical pulses by explosive compression of a magnetic field," Pribory dlya Nauchn. Issled., No. 11 (1967).
7. J. C. Crawford and R. A. Damerow, "Explosively driven high-energy generators," J. Appl. Phys., 39, No. 11 (1968).